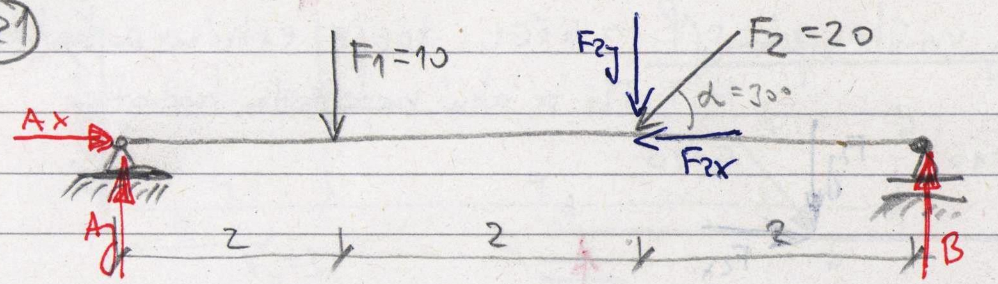
VNÚT. SILY - JEDNODUCHÝ NOSNÍK (F_1, F_2)

PR1



$F_1 = 10\,kN$
$F_2 = 20\,kN$
$\alpha = 30°$

1. stat. určitosť: je úloha riešiteľná iba pomocou podm. rovnováhy?

$n = n_v - n_o = 3° - (2+1) = 3° - 3° = 0°V$ - stat. určitý problém

2. Väzbové reakcie:

$\sum F_{ix} = 0 : A_x - F_{2x} = 0 \rightarrow A_x = F_{2x} \rightarrow \boxed{17{,}32\,kN = A_x}$

$F_{2x} = F_2 \cdot \cos 30° = 17{,}32$

$F_{2y} = F_2 \cdot \sin 30° = 10\,kN$

$\sum F_{iy} = 0 : A_y - F_1 - F_{2y} + B = 0$

$\sum M_{iA} = 0 : -F_1 \cdot 2 - F_{2y} \cdot 4 + B \cdot 6 = 0$

$B = \frac{F_1 \cdot 2 - F_{2y} \cdot 4}{6} = \frac{20+40}{6} \rightarrow \boxed{B = 10\,kN}$

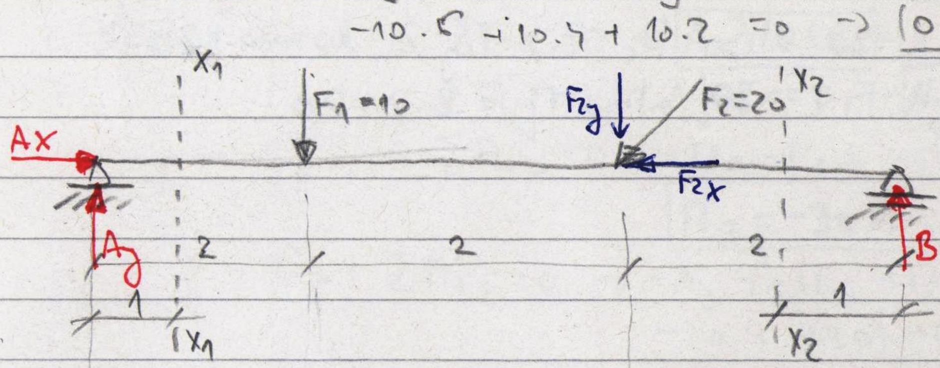
$A_y = F_1 + F_{2y} - B$

$A_y = 10 + 10 - 10 \Rightarrow \boxed{A_y = 10\,kN}$

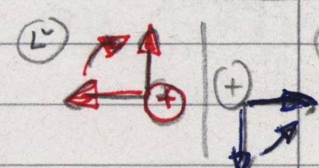
k.p.:

$\sum M_{iB} = 0 : -A_y \cdot 6 + F_1 \cdot 4 + F_{2y} \cdot 2 = 0$

$-10 \cdot 6 + 10 \cdot 4 + 10 \cdot 2 = 0 \rightarrow \boxed{0 = 0}$



MUSÍ BYŤ ZNAM. DOHODA!



3. Vnútorné sily v ľub. mieste (priereze) telesa: $x_1 = 1,\ x_2 = 5$

$x_1 :\ N_{x_1} = \sum F_{ix}^{(Ľ)} = \sum F_{ix}^{(P)} \rightarrow N_{x_1} = \sum F_{ix}^{(Ľ)} = -A_x = -17{,}32\,kN$

alebo $N_{x_1} = \sum F_{ix}^{(P)} = -F_{2x} = -17{,}32\,kN$

$T_{x_1} = \sum F_{iy}^{(Ľ)} = \sum F_{iy}^{(P)} \rightarrow T_{x_1} = \sum F_{iy}^{(Ľ)} = A_y = 10\,kN$

alebo $T_{x_1} = \sum F_{iy}^{(P)} = +F_1 + F_{2y} - B = 10 + 10 - 10 = 10\,kN$

$M_{o\,x_1} = \sum M_{ix_1}^{(Ľ)} = \sum M_{ix_1}^{(P)} \rightarrow M_{o\,x_1} = \sum M_{o\,x_1}^{(Ľ)} = A_y \cdot 1 = 10\,kNm$

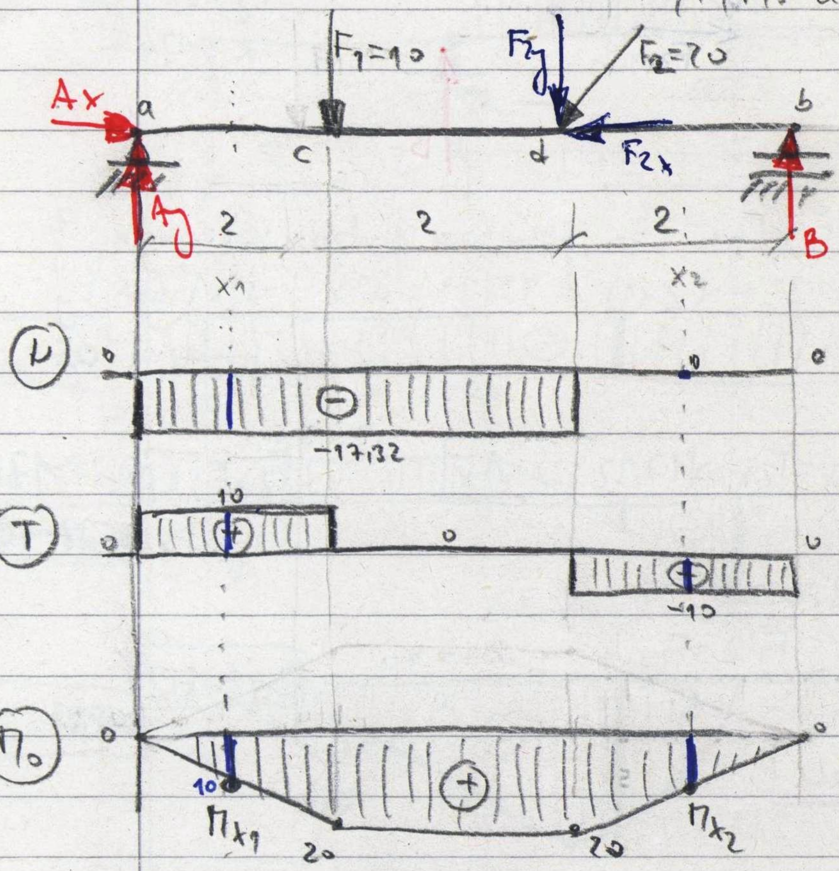
alebo $M_{o\,x_1} = \sum M_{o\,x_1}^{(P)} = -F_1 \cdot 1 - F_{2y} \cdot 3 + B \cdot 5 = 10\,kNm$

$x_2 :\ N_{x_2}^{(P)} = 0 \ \wedge \ N_{x_2}^{(Ľ)} = -A_x + F_{2x} = -10 + 10 = 0$

$T_{x_2}^{(P)} = -B = -10\,kN \ \wedge \ T_{x_2}^{(Ľ)} = A_y - F_1 - F_{2y} = 10 - 10 - 10 = -10\,kN$

$M_{o\,x_2}^{(P)} = B \cdot 1 = 10\,kNm \ \wedge \ M_{o\,x_2}^{(Ľ)} = A_y \cdot 5 - F_1 \cdot 3 - F_{2y} \cdot 1 = 50 - 30 - 10 = 10\,kNm$

4. Priebehy vnútorných síl → ÚČEL: kde je extrémna hodnota N, T, M_o a akú nadobudne hodnotu



$N_a^{TĽ} = 0$, $N_a^{TP} = -A_x = -17{,}32$

$N_c = N_a^{TP} = N_d^{TĽ} = -17{,}32$

$N_d^{TP} = -A_x + F_{2x} = -17{,}32 + 17{,}32 = 0$

$N_b = N_d^{TP}$

$T_a^{TĽ} = 0$; $T_a^{TP} = A_y = 10$

$T_c^{TĽ} = T_a^{TP} = 10$; $T_c^{TP} = A_y - F_1 = 0$

$T_d^{TĽ} = T_c^{TP} = 0$; $T_d^{TP} = A_y - F_1 - F_{2y} =$

$= 10 - 10 - 10 = -10$

$T_b^{TĽ} = T_c^{TP} = -10$; $T_b^{TP} = 0$

pretože $A_y - F_1 - F_{2y} + B = 0$

vzhľadom ku kĺbovému spojeniu je veličina nulová

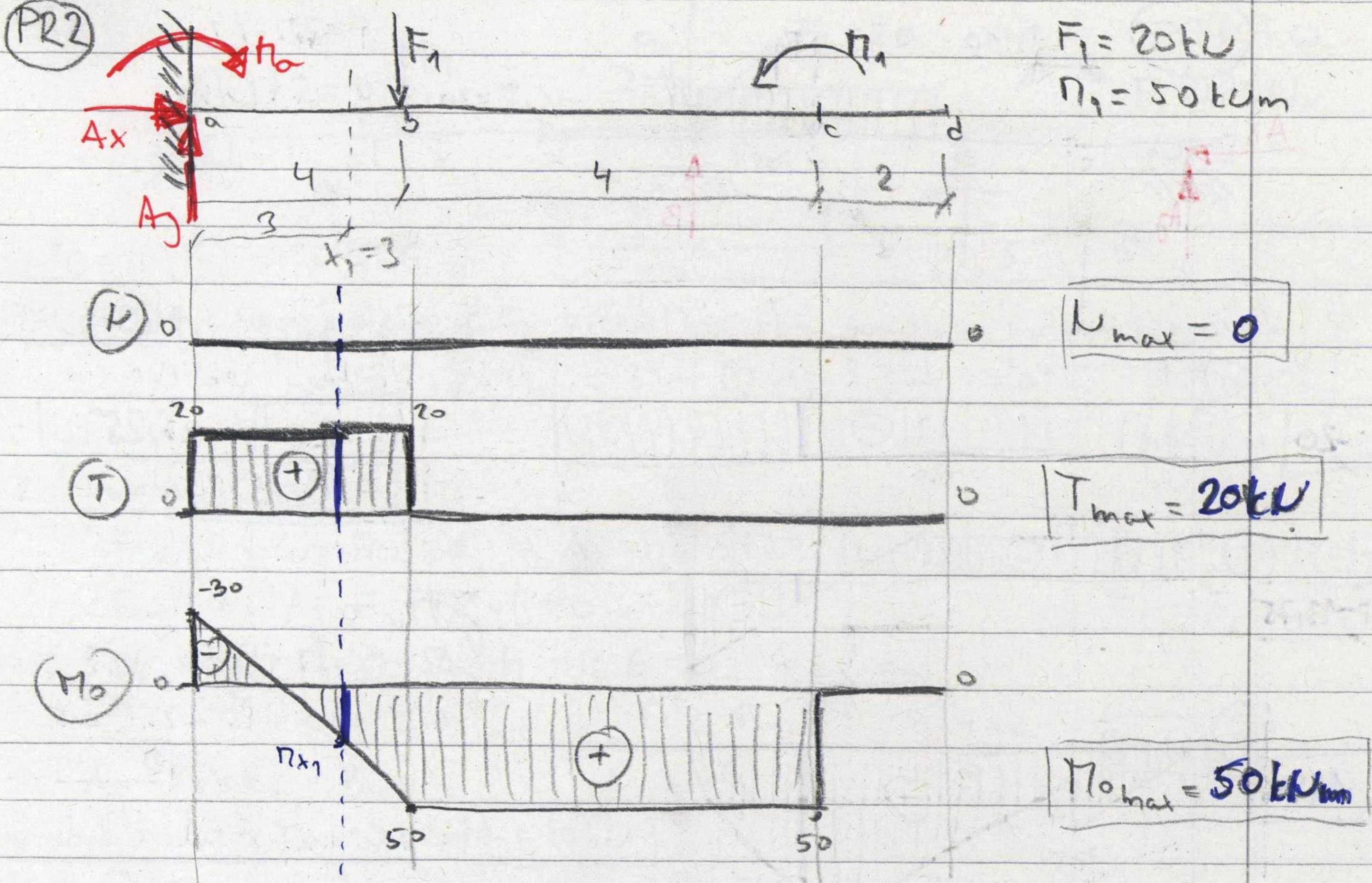
$M_a = 0$

$M_c = A_y \cdot 2 = 20$ alebo $M_c = B \cdot 4 - F_{2y} \cdot 2 = 40 - 20 = 20$

$M_d = A_y \cdot 4 - F_1 \cdot 2 = 20$ alebo $M_d = B \cdot 2 = 20$

$M_b = 0$

VNÚTORNÉ SILY - KONZOLA (F_1, M_1)



1. Stat. určitosť : $n = n_v - n_o = 3° - 3° = 0°$ - úloha stat. určitá
2. Väzbové reakcie : $\sum F_{ix} = 0 \rightarrow \boxed{A_x = 0 kN}$

$\sum F_{iy} = 0 \rightarrow A_y - F_1 = 0 \rightarrow \boxed{A_y = F_1 = 20 kN}$

$\sum M_{ia} = 0 \rightarrow -M_a - F_1 \cdot 4 + M_1 = 0 \rightarrow M_a = M_1 - F_1 \cdot 4 = 50 - 80$

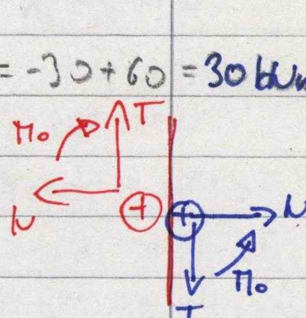
$\boxed{M_a = -30 kNm}$

k.p: $\sum M_{ib} = 0 \rightarrow -A_y \cdot 4 - M_a + M_1 = 0$

$-20 \cdot 4 - (-30) + 50 = 0 \rightarrow \boxed{0 = 0}$

3. Vnút. sily v priereze $x_1 = 3$

$N_{x_1} = -A_x = 0$; $T_{x_1} = A_y = 20 kN$; $Mo_{x_1}^{(ľ)} = M_a + A_y \cdot 3 = -30 + 60 = 30 kNm$



4.

$N_a^{TĽ} = 0$, $N_a^{TP} = -A_x = 0$; $N_b = 0$, $N_c = 0$, $N_d = 0$

$T_a^{TĽ} = 0$; $T_a^{TP} = A_y = 20$

$T_b^{TĽ} = T_a^{TP} = 20$; $T_b^{TP} = A_y - F_1 = 20 - 20 = 0$

$T_c = T_b^{TP} = 0$; $T_d = T_c = 0$

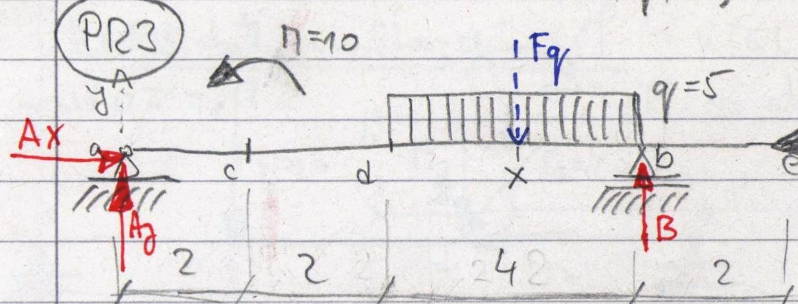
$M_a = M_a = -30 kNm$

$M_b = M_a + A_y \cdot 4 = -30 + 80 = 50 kNm$

$M_c^{TĽ} = M_a + A_y \cdot 8 - F_1 \cdot 4 = -30 + 160 - 80 = 50 kNm$

$M_c^{TP} = M_c^{TĽ} - M_1 = 50 - 50 = 0 kNm$

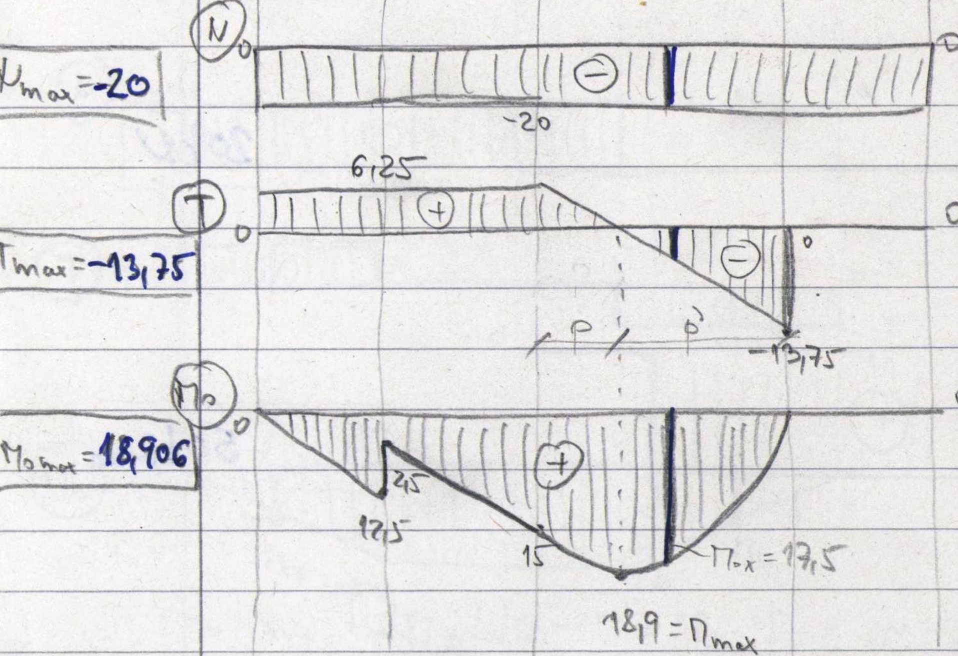
VNÚTORNÉ SILY (F, q, M)



$F = 20\,kN$

$q = 5\,kN/m$

$M = 10\,kNm$



1. $n = n_v - n_o = 3 - (2+1) = 0°$ s.u.
2. $F_q = q \cdot 4 = 5 \cdot 4 = \boxed{20\,kN = F_q}$
3. Väzbové reakcie:

$\boxed{A_x = 20}$ $\boxed{A_y = 6{,}25\,kN}$

$\boxed{B = 13{,}75\,kN}$

k.p.: napr. k bodu e

$\Sigma M_{ie} = 0:$

$-A_y \cdot 10 + M + F_q \cdot 4 - B \cdot 2 = 0$

$-62{,}5 + 10 + 80 - 27{,}5 = 0$

$\boxed{0 = 0}$

4. Vnútorné sily v bode x

$x = 6\,m$

$N_x^{(L)} = -A_x = -20,\quad N_x^{(P)} = -F = -20$

$T_x^{(L)} = A_y - q \cdot 2 = 6{,}25 - 10 = -3{,}75$

$T_x^{(P)} = -B + q \cdot 2 = -13{,}75 + 10 = -3{,}75$

$M_x^{(L)} = A_y \cdot 6 - M - (q \cdot 2) \cdot 1 = 37{,}5 - 10 - 10 = 17{,}5$

$M_x^{(P)} = B \cdot 2 - (q \cdot 2) \cdot 1 = 27{,}5 - 10 = 17{,}5$

5. Priebehy vnútorných silových veličín:

N_a [TL: 0; TP: $\Sigma F_{ix}^{(L)} = -A_x = -20$]

$N_c = N_d = N_b = N_A = -20$

N_e [TL: -20; TP: $-A_x + F = 0$]

T_a [TL: 0; TP: $A_y = 6{,}25$]

$T_c = T_d = T_a = 6{,}25$

T_b [TL: $A_y - F_q = 6{,}25 - 20 = -13{,}75$; TP: $A_y - F_q + B = 0$]

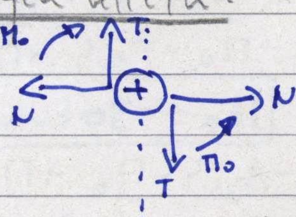
$M_a = 0$

M_c [TL: $A_y \cdot 2 = 12{,}5$; TP: $A_y \cdot 2 - M = 2{,}5$]

$M_d = A_y \cdot 4 - M = 15$

$M_b = A_y \cdot 8 - M - F_q \cdot 2 = 50 - 10 - 40 = 0$

$M_e = 0$ pre vykreslenie paraboly treba tretí bod, napr. $x \to M_x = 17{,}5$



6. Nebezpečný prierez x_N:

$x_N:\ \frac{T_d}{q} = p \to p = \frac{6{,}25}{5} = 1{,}25$

$x_N = 2 + 2 + 1{,}25 = 5{,}25\,m$

resp. $\frac{T_b}{q} = p' \to p' = \frac{-13{,}75}{5} = -2{,}75$

$x_N = (2+2+4) + p' = 8 - 2{,}75 = 5{,}25\,m$

7. EXTRÉMY VNÚT. SIL. VELIČÍN:

$N_{max} = -20\,kN$

$T_{max} = -13{,}75\,kN = T_b$

$M_{o\,max} = M_{o(x_N)} = B \cdot 2{,}75 - (q \cdot 2{,}75) \cdot \frac{2{,}75}{2}$

$M_{o\,max} = 18{,}906\,kNm$